**Stochastic-Based Analysis of Availability of Guns and Violent Mass Shootings**

Intro to Mathematical Modelling

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# Introduction

## Background Information

Gun control in the U.S. has become a heavily contested debate in the wake of growing instances of mass shootings and gun violence (Anisin, 2018, p. 55). “A mass shooting is often assumed to contain the presence of the following characteristics: (1) the death of four or more people; (2) a killing that is carried out by a lone shooter; (3) an episode of killing, not separate incidents of killing; and (4) the act must not be related to gang activity or robbery,” cites the *Journal of Comparative and Applied Criminal Justice* (Anisin, 2018, p. 55). These “people” can either be armed or unarmed depending on several factors including: gun laws (including purchase and carry laws), the role and experience of the “civilian” (i.e. security guard, policeman, ex-military bystander, etc.), and several other complex factors. This paper will focus on analysing the availability of guns in society as a function of gun laws and complex sociological factors that enable a fraction of society to buy a weapon. There is contention between the effect of this gun availability and its effect on mass shooting outcomes. It is argued that right-to-carry laws lead to more mass shootings over a period via greater availability of guns in society (Duwe, Kovandzic, & Moody, 2002). However, it is also argued that greater access to guns provides “people” in attacks with a method of personal defence. This paper stochastically simulates mass shootings based on an elementary mathematical model and stochastic Markov process to analyse the effects of gun availability on aggregated mass shooting deaths over time.

## Mathematical Approach from Literature

The simulation this paper builds and analyses extends the study: *Dependence of the firearm-related homicide rate on gun availability: a mathematical analysis*’s propagated gun-homicide violence and one-against-many attack literature (Wodarz & Komarova, 2013). The paper proposes both an elementary model for propagated gun-homicide violence based on gun availability and a stochastic simulation of the outcome of mass shootings using a Markov Process.

1. ,

where F(g) = overall risk of death, z(g) = number of armed offenders, f(g) = probability of a person to die during an attack

1. where h is the illegal availability of guns in society

“We suppose that there are http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e114 people within the range of a gunshot of the attacker, and http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e115 of them are armed. At each time-step, the state of the system is characterized by an ordered triplet of numbers, http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e116, where http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e117 tells us whether the attacker has been shot down (http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e118) or is alive (http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e119), where http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e120 is the number of armed people in the crowd, and http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e121 is the number of unarmed people. The initial state is http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e122. At each time-step, the attacker shoots at one person in the crowd (with the probability to kill http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e123), and all the armed people in the crowd try to shoot the attacker, each with the probability to kill http://journals.plos.org/plosone/article/file?type=thumbnail&id=info:doi/10.1371/journal.pone.0071606.e124.” The Markov process then becomes:

We will use these equations to formulate this paper’s extended simulation.

# Model Extension

## Overview

The model was first extended by introducing stochastic processes and functions to simulate: 1) An individual mass shooting given a gun availability value and n civilians, 2) A year of mass shootings given a gun availability in a society and an initial attack. These two stochastic processes were extended from the previous equations (1) – (8) cited in the previous sections. The goal was to determine if there is an optimized g value which minimizes deaths within an individual attack and over a time-dependent series of attacks in society.

## Individual Stochastic Mass Shootings

Before the Markov process can take place, the simulation must determine the proportion of armed and unarmed attackers and the d and p probabilities for attackers and armed civilians.

1. , where a(g) is armed attacker ratio function

The Markov process presented earlier is then used to simulate the attack and calculate a function for f(g). The simulation is dependent on the status of the state vectors not reaching an end-state (i.e. attacker is dead or all civilians are dead) and a tfinal which we assume to be when the attack ends due to police response. The discussion of where these equations derive from resides below in the assumptions and simulation parameters section. First, (1) is redefined to be in terms of total deaths in an attack instead of the probabilities of death in an attack. It is also extended to include a function of time so that subsequent attacks in society are a product of integrating previous attack deaths (i.e. an inherent ODE). This equation is shown below with an initial value (initial attack) given at t0.

Equation (2) is used from literature to determine the gun availability in society to criminals. The h constant is also derived from Wodarz and Komora’s model and analysis. Equation (14) is a recasting of (1) to include time-dependency where F is now cumulative civilian causalities in society and f is individual attack civilian casualties.

## Assumptions & Simulation Parameters

### Assumptions

The n value was linearly varied between 5 and 50 civilians. The lower limit, 5, was chosen as the number of civilians (n) that would ‘normally’ result in a classified mass shooting (at least 95%) of the simulations; given that a mass shooting must have at least 4 victims (Anisin, 2018). The upper limit was chosen based on the *Journal of Comparative and Applied Criminal Justice* configuration of the normal distribution (upper 95%) of the number of victims in an attack (including deaths and injuries).

It was assumed that there would be a linear distribution of civilians present at the attack. This assumption is a possible extension of the model and could be better analysed and modelled to provide a more accurate picture.

The space of attacks is enclosed so that we assume civilians do not disperse as the attack goes on. This could mean movie theatres, classrooms, concert halls, etc.

### Parameters

The following parameters were assigned for the individual attack simulation functions (9)-(11)

0.2

For d(n) from (8) the attacker’s probability to kill is calculated based on a logistical growth model for the number of civilians present during the attack. The function fits the literature and expectation of what would happen in an individual attack at each moment. At relatively low civilian numbers (n=5), the attacker’s probability of killing one of the civilians will be low and the relative rate of change will be low. The rate of change of probability of attacker killing civilian increases to a point of inflection of (n=25). This can be explained by several factors and is a potential further extension of the model. Frantic pedestrian traffic modelling, psychological spraying vs. choice of victim, and other factors could be considered for more rigorous extension and modelling of simulated attacks. Lastly, as the number of civilians reaches a relatively high number (n>40), the attacker’s probability will flatten out at an asymptotic limit. The specific parameters for the model were estimated based on probability outcomes that would lead to attacks lasting for a duration close to tfinal.  For d(g) from (8) the attacker’s probability to kill is calculated based on a sigmoid function for the availability of guns present during in the society the attack occurs in. This mirrors the effect of a society with greater availability of guns. At g=0, there will be relatively low amounts of gun training as there is no access to possession of guns in society. Therefore, the attacker is likely to be untrained and thus have a relative low deadliness. As the g value increases, there will be a point of inflection around g=0.5 as guns reach their way into mainstream culture of the society and there is more associated training and ‘practice’. This will continue to increase as g approaches 1, however the rate of increase will be relatively less as the society gets closer to 1.

0.05

For p(g) from (9) the armed civilian’s probability to kill is calculated based on a sigmoid function for the availability of guns present during in the society the attack occurs in. This mirrors the effect of a society with greater availability of guns. At close to g=0, there will be relatively low amounts of gun training as there is no access to possession of guns in society. Therefore, the armed civilians are likely to be untrained and thus have a relative low deadliness in their shots. As the g value increases, there will be a point of inflection around g=0.5 as guns reach their way into mainstream culture of the society and there is more associated training and ‘practice’. This will continue to increase as g approaches 1, however the rate of increase will be relatively less as the society gets closer to 1. The µ value scales g(n) to decrease the range of probabilities to realistic values. This accurately reflects the probability of a successful shot by the armed attacker given a hectic situation with many outstanding factors outside of ballistic aim skill.

(Wodarz & Komarova, 2013) This is based on American society data from literature. This assumption could however be a possible extension of the model, as the fraction of those who take up (and carry) arms is most likely non-linear across g values.

(Wodarz & Komarova, 2013)

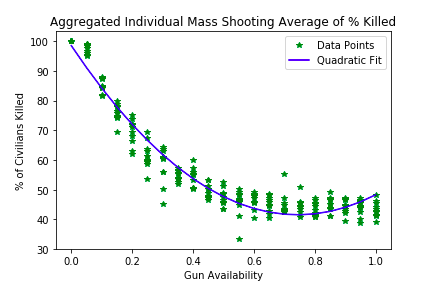
### Discussion of Randomness

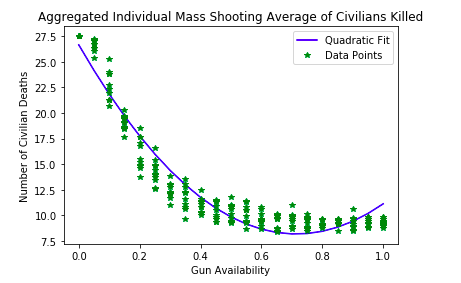
The stochastic Markov process includes a crucial factor of randomness at each step in the attack. This is both essential to the stochastic process itself (to account for the lack of epistemological data) and is a limiting factor in the accuracy of the simulated-derived equation - f(g).

# Results

## Stochastic Individual Mass Shootings

The simulation of attacks begins. The stochastic model is run given fitted parameters that align with qualitative literature on mass shootings. The stochastic model averages a range of simulation of attacks from n=5 to n=50, 10 trials at each n value. This is run over 20 different g values from g=0 to g=1 to produce the two graphs below. Each time the code is run, different values are outputted because of the inherent stochastic randomness. However, the same quadratic trend is observed in each case.





The quadratic nature of the gun availability to death outcome of the mass shooting reaches an optimized minimum at g=0.75. The trade-off between armed civilians and the power of the attacker is displayed here. The associated quadratic fit from this data yields the below equation for f(g).

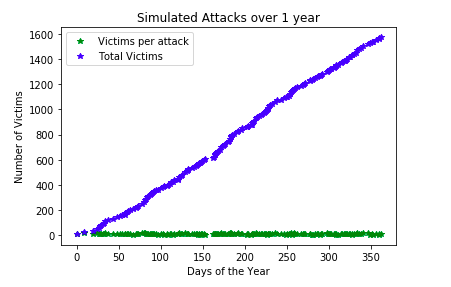
## Analytic Optimization of Time-Dependent Mass Shootings

1. = 0, solve for g

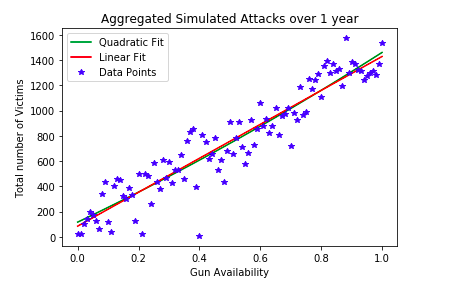
Given that g<0, there is no analytic value from (14) our master gun violence equation as shown in (17) which optimizes the gun availability in society. We therefore must test the values at the limits of our range g=0 and g=1 to find our optimized total death toll. This is also a possible extension of the model as we could design a function (14) which reaches an optimized value between g=0 and g=1.

## Stochastic Simulation of Time-Dependent Mass Shootings

Lastly, this paper simulates time-dependent, discrete mass shootings over 1 ‘year’ in societies with differing g values. This compares the analytic hypothesis from section 3.2 with stochastic-generated data. The first graph below shows the victim count for attacks that occur throughout a ‘year’ in a society with g=1. The stochastic process begins with an initial spawned attack at t0 and then calculates at each time step (days) the probability of a subsequent attack occurring. This probability is dependent on the availability of guns to attackers, z(g) and an activation function for ∫f(g). The activation should include both a deterrent function, Ω(x), where the input is the proportion of people the attacker kills before dying himself (∫f(g)/n), and the cumulative number of people who have died, which is ∫f(g).



The below graph, shows the aggregated simulated attacks for different gun availability values in different societies fit to linear and quadratic lines.



# Conclusion

## Analysis

The model began with an inquiry into the relationship between gun availability and the death toll in mass shootings. First, we simulated individual attacks given a set of assumptions for how the gun availability affects the number of armed citizens and the deadliness of attackers and armed citizens. We then cumulated the average death tolls of these attacks for multiple simulations to identify a pattern. This yielded a quadratic relationship, with an optimized value at g=0.75 for the least number of deaths. This f(g) equation was then analytically used in F(g) for a fixed time to optimize the g value over the interval 0 to 1. However, it was found that the only solution was negative and therefore we must check the endpoints g=0 and g=1 for the least death toll outcome for cumulative mass shootings.

Lastly, the second simulation approximated this value with an activation sigmoid function to discretely decide if attacks occur at each time step. As the gun availability rose the number of attacks also rose linearly. Thus, g=0 is the optimal value for the least attacks in society, given the model’s assumptions.

## Limitations of Model & Future Developments

The model is limited by the accuracy of its assumption. This is preliminary attempt to simulate individual attacks in a stochastic way and then discretize these attacks through time in a society to track the long-term effects of gun availability on death toll for mass shootings. In a general sense, these findings are reflected by the difference between say American (g ≈ 0.6) and Australian (g ≈ 0.1) societies. However, the model lacks rigorous comparative analysis to societal functions. This could be studied by taking mass shootings by specific country or state, where g values vary.

Furthermore, the functions used in the individual simulations: (10) and (11) are not fitted with measured data. Therefore, the assumed values the model uses for the simulation are nothing more than an educated guess. This is a major limitation and opportunity for future development. The field of mathematically modelling mass shootings and gun violence is relatively underdeveloped. There is a large need for empirical data to fit the generated models.

Lastly, the analytical optimization analysis in section 3.2, with equation (16) and (17) assumes a continuous property of mass shooting deaths, when this is not the reality. For the sake of analytical derivation, this was assumed. However, a model that discretized the mass shootings per time, would present a more accurate reflection of reality and perhaps yield an optimized value in the interval (0,1). This model would also include an ‘activation function’ that included the deterring affect of armed civilians (as g grows larger) and the psychological reliance on guns in mass shootings as an ideological act against groups of people or society as a whole. This activation function should be empirically determined. Determining that alone would be a complex extension to the model itself.

# References

## Literature

Anisin, A. (2018). A configurational analysis of 44 US mass shootings: 1975–2015. *International Journal of Comparative and Applied Criminal Justice, 42*(1), 55-73. doi:10.1080/01924036.2016.1233444

Duwe, G., Kovandzic, T., & Moody, C. E. (2002). The impact of right-to-carry concealed firearm laws on mass public shootings. *Homicide Studies, 6*(4), 271-296.

Wodarz, D., & Komarova, N. L. (2013). Dependence of the firearm-related homicide rate on gun availability: a mathematical analysis. *PloS one, 8*(7), e71606.

# Appendix

## Python Simulation Code

Uploaded to GitHub at <https://github.com/jaisal1024> and available by email